

Gravity Role in Classical Electrodynamics of Charged Point Source

M.B.Golubev*

Russian Federal Nuclear Center - All-Russian S&R Institute of Experimental Physics

Abstract

This paper deals with the problem of a point-like charged source under the influence of the external electromagnetic field in terms of perturbation theory for GR equations. It is obtained that GR, in contrast with the classical electrodynamics, in linear perturbation theory predicts an unlimited growth of the dipole perturbation. It is shown that the reason for this unlimited perturbation growth might be related to the presence of the unstable rotational perturbation mode. The analysis of the conditions under which this instability may disappear is performed. The momentum value at which the stability is reached is estimated. These estimations give the electron spin by the order of magnitude (when charge value is equal to elementary one).

Introduction

The gravitation force is commonly considered negligible in elementary particle interaction since proton-electron gravitation interaction force is forty orders of magnitude less than that of electromagnetic interaction. On the other hand, Einstein's equation system is closed, while Maxwell's equations requiring closing by sources motion equations. In other words, within the framework of general relativity it is possible to pose a problem of an electromagnetic field source (a charged point source or a charged black hole) motion and emission under the influence of an external electromagnetic field. So while it is necessary to postulate Lorenz's force in electrodynamics, it is possible to obtain it using general relativity.

This statement does not look like a paradox if one takes into account the following fact. It is possible to postulate in electrodynamics not Lorenz's force but the energy conservation law. In this case it is possible to obtain Lorenz's force in electrodynamics as well. Einstein's equations provide for energy conservation automatically because of Bianchi's identity. So it is not at all surprising that they should also contain field sources equations of motion (Lorenz's force). However, there is no ground to expect *a priori* that a field source equations of motion will be the same in electrodynamics and in GR. More than that, physical mechanisms leading to a source acceleration are basically different under electrodynamics and GR. In electrodynamics the acceleration is a result of electromagnetic field interaction with the charge (by Lorenz's force). Since for a pointlike charge the field at the point of charge has infinite intensity, Lorenz's force for a pointlike charge is ill - defined and requires a renormalization. As a result, non-physical runaway solutions exist in electrodynamics. In GR the acceleration emerges as a result of the external field interaction with the source own field in some vicinity of the source. This interaction contributes to the electromagnetic field energy resulting in metrics curvature that is interpreted as the acceleration by an external observer.

Since the right-hand side of Einstein's equations $G^{\mu\nu} = \kappa T^{\mu\nu}$ contains the gravitation constant as a multiplier to the interaction energy while the left-hand side contains the metric tensor derivatives (including acceleration), one may agree that the acceleration should then be proportional to the gravitation constant. Let's show how a small quantity - gravitation constant - falls out from the expression for the acceleration. Let $g_{\mu\nu}^0 + h_{\mu\nu}$ be a charged source metrics perturbed by the external field. The main term of the nonperturbed part is $g_{00}^0 = 1 - \kappa mc^2/r + \kappa Q^2/r^2$ where m is the mass and Q is the charge. The main term of the perturbed part is $h_{00} = 2\vec{a}\vec{r}/c^2$, where \vec{a} is the acceleration. The main term by the powers of r in the expression for the linear by the perturbation part of the Einstein tensor will be: $\partial g_{00}^0 \partial h_{00} \sim \kappa mc^2/r^2 - a/c^2$. The corresponding term of the right-hand side of the linearized Einstein's equations constitutes the energy of the external field interaction with the source own field

*e-mail: mikhailg@albatross.md08.vniief.ru

multiplied by gravitation constant: $\kappa T_{00} \sim \kappa Q/r^2 E$. So the expression: $\kappa mc^2/r^2 \sim a/c^2 \sim \kappa Q/r^2 E$ results in $a \sim QE/m$.

So it is clear it's sufficient to consider the first order perturbation theory equations in order to study a charge motion dynamics in GR. One of most advanced perturbation techniques for GR - Einstein-Infeld-Hofman's procedures was used by Anderson [1] To obtaine not only Lorenz's force but radiation reaction force as well. However, Einstein-Infeld- Hofman's technique uses decomposition by the powers of a few parameters one of them usually being the charge. This limits the above technique to the distances much greater than the classical radius (Q^2/mc^2), since at the classical radius the contribution of the charge into the metrics becomes equal to that of the mass. Below we will show that at this distances of the order of classical radius GR gives the results that are substantially different from those of classic electrodynamics. Bicak [2] in 1980 studied the problem of a charged black hole motion in a constant asymptotically uniform electric field using perturbation theory linear by the amplitude. Acceleration obtained by Bicak was exactly the same as the Lorenz's force. This paper and paper [3] extends Bicak's technique to a charged point source and time-dependent spatially non-uniform perturbation.

Problem Statement and Gage-Invariant Perturbation Theory Equations

Let's place coordinate center at the charge center and consider axially symmetric perturbations of the first order by amplitude magnitude. Axially symmetric perturbations for spherically symmetric metrics as shown by Regge and Wheeler [5] permit variables separation, i.e. separation of the angle-dependent part using spherical harmonics. In the process the equations split into the equations for polar perturbations (even ones) and for axial perturbations (odd ones). Accelerated reference systems have in metric coefficient g_{00} a term like $2arcos\theta$, where a is acceleration. This fact is consistent with the equivalence principle, i.e. in the accelerated charge rest reference system we have a uniform gravitational field. So a charge motion dynamics is defined by the first spherical harmonic for polar perturbations and we will look for the solutions of the equations for radial functions corresponding to the first spherical harmonic of polar perturbations. The initial unperturbed metrics is described by the Reisner-Nordstrom solution:

$$\begin{aligned} ds^2 &= \Delta/r^2 dt^2 - r^2/\Delta dr^2 - r^2(\sin^2 \theta d\phi^2 + d\theta^2), \\ \Delta &= r^2 - 2mr + Q^2, \end{aligned} \quad (1)$$

where we assume $c = \kappa = 1$. Using these units electron mass and charge are respectively:

$$m_e = 6.67 \cdot 10^{-56} \text{ sm}, e = 1.38 \cdot 10^{-34} \text{ sm}$$

Reisner-Nordstrom solution describes a charged black hole when $Q < m$. When $Q > m$ horizons disappear and we have a naked singularity. We will be interested in exactly this case since in a classical black hole metrics a classical radius is under the horizon but the most interesting things take place exactly at the distances of the order of the classical radius. Metrics perturbation corresponding to the first spherical harmonic of polar perturbations is represented in the following expression:

$$\begin{aligned} h_{\mu\nu} &= \begin{pmatrix} h_{00} \cos \theta & h_{01} \cos \theta & 0 & -h_{03} \sin \theta \\ h_{01} \cos \theta & h_{11} \cos \theta & 0 & -h_{13} \sin \theta \\ 0 & 0 & h_{22} \cos \theta \sin^2 \theta & 0 \\ -h_{03} \sin \theta & -h_{13} \sin \theta & 0 & h_{22} \cos \theta \end{pmatrix} \\ A_\mu &= (A_0 \cos \theta, A_1 \cos \theta, 0, -A_3 \sin \theta) \end{aligned} \quad (2)$$

Linear perturbation theory equations are reduced to a single wave equation for a gage-invariant (independent of infinitely small transformations $x^{\mu'} = x^\mu + \xi^\mu$, where $\xi^\mu = (\xi^0(r, t) \cos \theta, \xi^1(r, t) \cos \theta, 0, -\xi^3(r, t) \sin \theta)$) function H_3 , that can be used to express all the metric coefficient and electromagnetic potentials:

$$-H_{3,tt} + H_{3,r^*r^*} - \frac{2\Delta}{r^2(r - \frac{2}{3}r_0)^2} \left(1 - \frac{2Q^2}{r^2} + \frac{16r_0Q^2}{9r^3} - \frac{4r_0^2Q^2}{9r^4} \right) H_3 = 0 \quad (3)$$

Where r^* is defined by $dr/dr^* = \Delta/r^2$, and $r_0 = Q^2/m$ is the classical radius. At that, in gage

$$h_{22} = h_{03} = A_3 = 0, \quad h_{11} = \frac{2r}{\Delta} h_{13} \quad (4)$$

the metric coefficients and electromagnetic 4-vector potential are the simplest:

$$\begin{aligned} h_{01} &= -H_{3,t} \\ h_{13} &= H_3 \\ h_{00} &= -\frac{2\Delta^2}{r^4} H_{3,r} + \left(\frac{\Delta^2 + 3Q^2\Delta}{r^5} - \frac{6\Delta Q^2}{r^4 r_0} + \frac{\Delta}{r^3} - \frac{4\Delta^2}{r^4(r - \frac{2}{3}r_0)} \right) H_3 \\ A_0 &= Q\Delta \left[\frac{2}{r^4} H_3 + \left(\frac{3}{2r^2 r_0} - \frac{1}{r^3} \right) H_{3,r} \right] \\ A_1 &= \frac{3Qr(r - \frac{2}{3}r_0)}{2\Delta r_0} H_{3,t} \end{aligned} \quad (5)$$

Equation (3) was obtained for the first time in paper [2]. In this paper the time-independent solution corresponding to the asymptotically uniform electric field E_0 has been found. The corresponding acceleration $a = Q/mE_0$ was exactly equal to Lorentz's force. If we apply this equation to a point source with the electron - like parameters and limit ourselves with the distances of order of an electron classical radius, we can set the gravitational terms (terms of order $\frac{Q^2}{r_0^2} \sim 10^{-40}$) to zero. At that, $\Delta \rightarrow r^2$, $Q^2 \rightarrow 0$ and equations (3), (5) will look as follows:

$$-H_{3,tt} + H_{3,rr} - \frac{2}{(r - \frac{2}{3}r_0)^2} H_3 = 0 \quad (6)$$

$$\begin{aligned} h_{00} &= -2H_{3,r} + \left(-\frac{4}{r - \frac{2}{3}r_0} + \frac{2}{r} \right) H_3 \\ h_{11} &= \frac{2}{r} H_3 \\ A_0 &= Q \left[\left(\frac{3}{2r_0} - \frac{1}{r} \right) H_{3,r} + \frac{2}{r^2} H_3 \right] \\ A_1 &= Q \left(\frac{3}{2r_0} - \frac{1}{r} \right) H_{3,t} \end{aligned} \quad (7)$$

Equations (6) are well known in electrodynamics. The equations for the electromagnetic potential radial functions look just this way after variables separation. The difference is only in the fact that coefficient pole is not in zero but at the two thirds of a classical radius. It may seem, though, that when the gravitational terms approaches zero we should get purely electrodynamic equations (though for an accelerated reference system). The gravitation disappeared and left an unexpected trail after itself just as the Cheshire Cat left its smile after itself [7]. We can write down the general solution of equation (6):

$$H_3 = (C_1 t + C_2) \left(r - \frac{2}{3}r_0 \right)^2 + \frac{C_3 t + C_4}{r - \frac{2}{3}r_0} + \quad (8)$$

$$+ f_1'(r - t) + \frac{f_1(r - t)}{r - \frac{2}{3}r_0} + f_2'(r + t) + \frac{f_2(r + t)}{r - \frac{2}{3}r_0} \quad (9)$$

If in (8) we set $C_2 = \frac{r_0}{3Q} E_0 = \frac{Q}{3m} E_0$, and set all other constants to zero, we will get asymptotically uniform electric field E_0 and the acceleration will be $a = \frac{Q}{m} E_0$. Bicak obtained the same results by finding the exact solution for equation (3). Notice, that there are no runaway solutions in (8),(9) that are so common for electrodynamics. Let's consider the wave solutions (9). They show unlimited growth when approaching the source and become infinite at the two thirds of the electron classical radius. This divergence cannot be removed by taking half a difference of incident and reflected waves as is usually done in electrodynamics since the pole in field and metrics expressions will not disappear. It is understandable that at some perturbation amplitude we have to take into account the contribution of the perturbation theory higher orders. Anderson in [1] performed decomposition by the charge powers. In our case it corresponds to the pole decomposition by the powers of r_0/r when

the singularity disappears and we will get radiation reaction force and runaway solutions related to it. It's evident that in reality the growing perturbation amplitude is limited at least by the intensity equal to that of an electron own field. When this intensity is reached it is necessary to take into account the contribution of perturbation theory higher orders that will become equal to the first order contribution. This way GR makes linear classical electrodynamics irrelevant when dealing with a charge motion dynamics and field description at the distances of the order of the classical radius.

Reasons for Singularity in Solution

It is useless to take into account the second and higher orders of perturbation theory if we do not understand the reason giving rise to the singularity in (9). By its behavior this singularity is similar to electromagnetic wave interaction with electrons in plasma when the wave approaches the critical density where it comes to resonance with Langmuir oscillations. But in this case there is no resonant frequency and it leads to conjecture about the interaction with some unstable mode, i.e. about the charged source metrics instability. The issue of black hole stability is a classical issue of perturbation theory. This issue is dealt with in papers by Regge - Wheeler [5] (singularity stability in Schwarzschild metrics), by Carter [6] (theorem on Kerr's metrics stability) and by Moncrief [4], who has considered the stability of Reisner-Nordstrom black hole. Moncrief has considered all spherical harmonics of the rotational and polar perturbations of Reisner-Nordstrom metrics for the black hole ($Q < m$) outside ($r > r_+$, where r_+ is the event horizon) and shown that there are no exponentially growing solutions. Still, there is one rotational mode, that he has not considered, and that can be unstable. Let the perturbation be like that:

$$\begin{aligned} h_{\mu\nu} &= \begin{pmatrix} 0 & 0 & h_{02} & 0 \\ 0 & 0 & h_{12} & 0 \\ 0 & 0 & 0 & 0 \\ h_{02} & h_{12} & 0 & 0 \end{pmatrix} \\ A_\mu &= (0, 0, A_2, 0) \end{aligned} \quad (10)$$

If we look for the solutions similar to $h_{\mu\nu} = e^{i\omega t} h_{\mu\nu}(r)$ $A_2 = e^{i\omega t} A_2(r)$ then the first order perturbation theory equations will look like follows:

$$\begin{cases} \omega^2 H_1 + H_{1,r^*r^*} + \frac{\Delta(6mr-4Q^2)}{r^6} H_1 - \frac{8\Delta Q}{r^5} A_2 = 0 \\ \omega^2 A_2 + A_{2,r^*r^*} - \frac{4\Delta Q^2}{r^6} A_2 + \frac{\Delta Q}{r^5} H_1 = 0 \end{cases} \quad (11)$$

$$H_1 = \frac{h_{02,r} - h_{12,t}}{r^3} - \frac{4QA_2}{r^4}$$

Following the way Moncrief used to prove Reisner-Nordstrom metrics stability, one can show, that some of ω^2 eigenvalues are necessary negative. That means the presence of exponentially growing solutions, i.e. instability. The physical properties of the unstable solution are interesting in relation to electromagnetic fields. In the unstable solution the magnetic field is directed along meridians and the electrical one along parallels. There is no angular dependency and Poincaré's vector is everywhere directed along the radius looking to the sphere with almost classical radius. So the energy is flowing into the potential well with the bottom located at the classical radius. To that end it is interesting to note that in metrics (1) the rest point of uncharged test particles is located at the classical radius. The attracting potential at the classical radius becomes repulsive, i.e. there really exists a gravitational potential well in metrics and this well seems to be the reason for this singularity. One can agree that this mode (10) has one significant drawback. It is singular along Z axis. That is, just like in case with Dirac's monopole there exists a thread along which the magnetic field is infinite. But if we take into account the higher orders of the perturbation theory this singularity will disappear. The open question is whether this mode will still be unstable. One more question is how will this mode transform if we turn on rotation and move from Reisner-Nordstrom solution to that by Kerr-Newman. The variable separation for Kerr-Newman solution is still unknown as well as its stability. At the same time even a solution with infinitely small rotation has a different topology. The singularity will change from point to ring-like. So it can be expected that the singular mode (11) will become regular. B. Carter (author of the famous Carter's theorem on Kerr's metrics stability) in his recent paper "Has the black hole equilibrium problem been solved?" [8] says that his theorem is related only to vacuum-type solutions and the issue of *electrovacuum* solutions is still open.

Conclusion

In conclusion it is shown that GR (in contrast with classical electrodynamics) in linear perturbation theory forecasts the unlimited growth of the dipole perturbation. This fact is a circumstantial evidence that the spherically symmetric solution for a charged source is unstable. It is natural to suppose that the potential well in wave equation (11) is related to the gravitational potential well of the metrics (1). There is an analogous potential well in Kerr-Newman metrics. Let's ask ourselves: at what momentum the potential well in Kerr-Newman metrics will be substantially different from that in the spherically symmetric Reissner-Nordstrom metrics? The analysis of test particles radial trajectories in Kerr-Newman metrics moving along the axis shows that the potential well is moving away from the center and becomes smaller in depth only when a (Kerr geometry parameter related to rotation) is greater than r_0 , when the momentum $M = mac > mr_0c = Q^2/c$. It leads to conclusion that if potential well flattening due to rotation can make the metrics stable it can happen only when momentum $M > Q^2/c$. The real electron momentum $M = \hbar/2 = 68.5 \text{ } e^2/c$. Since mass is absent in the expression for momentum, then according to this hypothesis proton momentum can differ from that of an electron at most in the nineteenth digit (the next term order of magnitude $\sim \frac{m}{Q} = \frac{\sqrt{\kappa} m_P c^2}{e} \sim 10^{-18}$).

Acknowledgements

Author would like to express his acknowledgement for useful discussions to M.V.Gorbatenko, V.V.Kassandroff, S.P.Kelner, V.I.Kogan, V.D.Shafranov and to O.N.Tazetdinov for this paper translation. I would like to express gratitude to my Teacher Vladimir Nikolaevich Likhachev.

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